# Subject: Leaving Certificate Maths

# Teacher: Mr Murphy

# Lesson 2: Functions

## 1.1 Learning Intentions

**After this week's lesson you will be able to;**

- Generalise a sequence
- Describe the nature of a sequence
- Represent a sequence in multiple ways
- Describe functions in terms of injective, surjective and bijective

## 1.2 Specification

| Student learn about | Student working at OL should be | In addition, students working at HL should be able to |
|---|---|---|
| 3.1 Numer systems | represent these numbers on a number line | ...y construct $\sqrt{2}$ and ... is nto rational |

| Students learn about | Students working at OL should be able to | In addition, students working at HL should be able to |
|---|---|---|
| **5.1 Functions** | – recognise that a function assigns a unique output to a given input<br>– form composite functions<br>– graph functions of the form<br>• $ax+b$ where $a,b \in \mathbf{Q}$, $x \in \mathbf{R}$<br>• $ax^2+bx+c$ where $a, b, c \in \mathbf{Z}$, $x \in \mathbf{R}$<br>• $ax^3+bx^2+cx+d$ where $a,b,c,d \in \mathbf{Z}$, $x \in \mathbf{R}$<br>• $ab^x$ where $a \in \mathbf{N}$, $b, x \in \mathbf{R}$<br>– interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions<br>– use graphical methods to find approximate solutions to<br>• $f(x) = 0$<br>• $f(x) = k$<br>• $f(x) = g(x)$<br>where $f(x)$ and $g(x)$ are of the above form, or where graphs of $f(x)$ and $g(x)$ are provided | – recognise surjective, injective and bijective functions<br>– find the inverse of a bijective function<br>– given a graph of a function sketch the graph of its inverse<br>– express quadratic functions in complete square form<br>– use the complete square form of a quadratic function to<br>• find the roots and turning points<br>• sketch the function<br>– graph functions of the form<br>• $ax^2+bx+c$ where $a,b,c \in \mathbf{Q}$, $x \in \mathbf{R}$<br>• $ab^x$ where $a, b \in \mathbf{R}$<br>• logarithmic<br>• exponential<br>• trigonometric<br>– interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions<br>– informally explore limits and continuity of |

## 2.3 Chief Examiner's

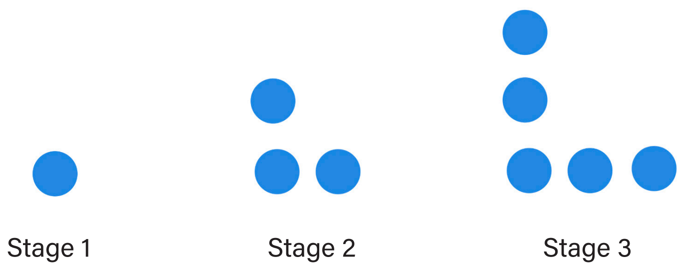
At Junior cycle, as referred to in section 1.1. The idea of a function cuts across all of thte syllabus strands, and might be profitably approached in an intregrated way rather than as a stand-alone-strand in itselft.

This is an important idea to keep in mind as you cover the course. Functions just like many other areas within mathematics are present across the syllabus and not just in one section. As functions is one of the largest sections that is present throughout the syllabus, we will start with it.

## 2.4 Functions Skills

Functions can be used as a way of investigating various situations in mathematics. They can allow us to look at current data, trends in data and therefore make predictions about the future. They can also help us to reverse an action to get back to some original conditions.

### Let's look at a simple sequence of dots:

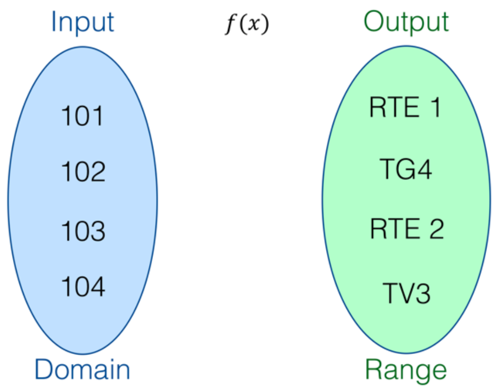


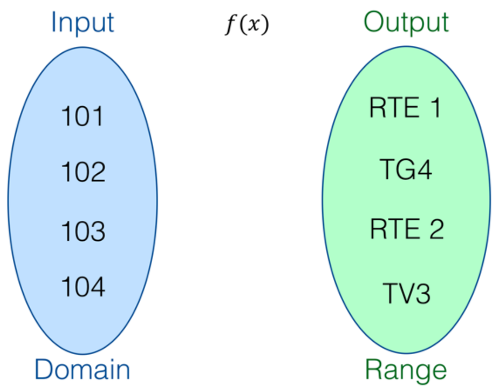

A useful strategy here is the idea of near, far, any. Can you figure out a stage that is near to these **(e.g. Stage 5)?** One that is far from them (e.g. Stage 50)? And then any stage, in other words could you come up with a rule that will help you figure out any stage. Write down some of your ideas below:

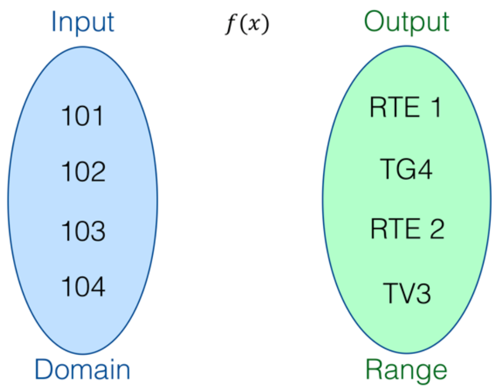
### Ideas:

This skill would have been used in patterns at Junior Cycle. These skills are as important at Senior Cycle. We can take this pattern and represent it in a number of ways, a table, a rule a rule etc. We will use all of these skills as we progress through our course. (refer to the video for week 2 for the completed analysis of this pattern).

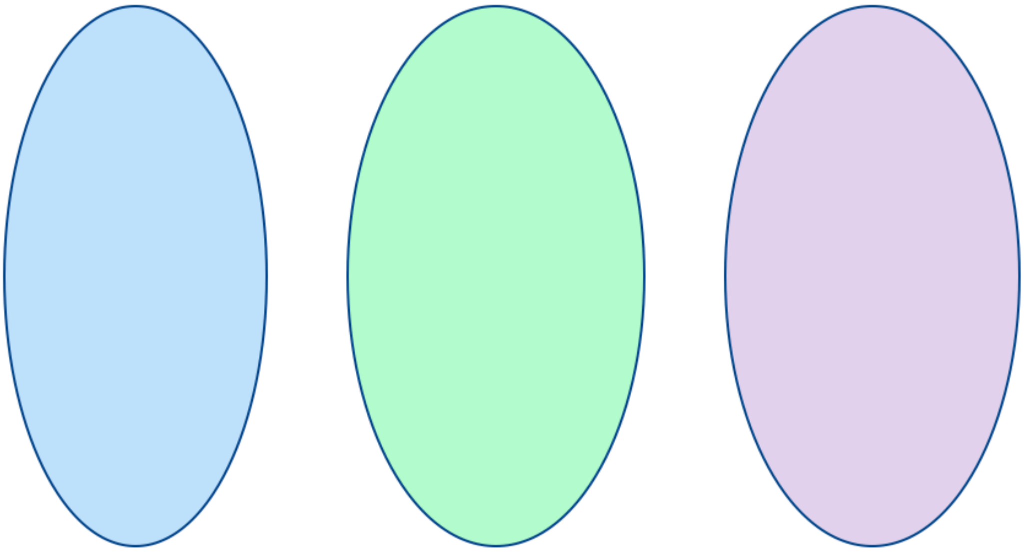
## 2.5 What makes a function?



## 2.6 Composite Functions



We can describe composite functions as a multi-layered function. In other words, it is a function on another function.

**Example:**

$$f(x) = 6x + 2 \qquad g(x) = x^2$$

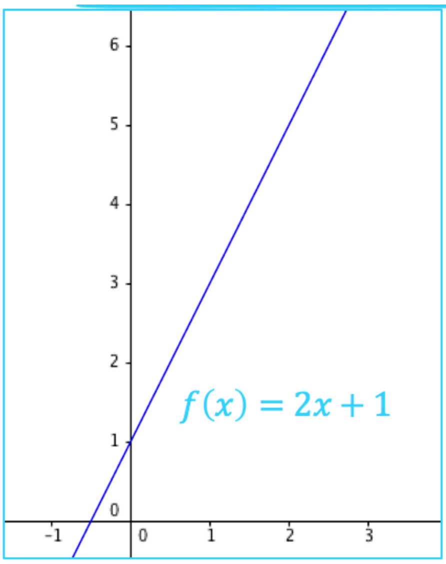
i) $f \circ g =$

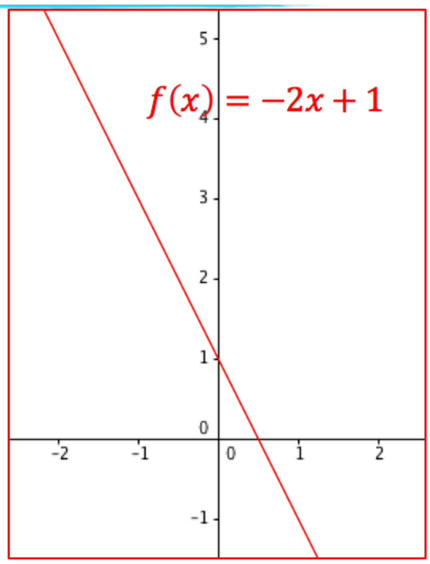
ii) $g \circ f =$

## 2.7 Types of Functions

### Linear:

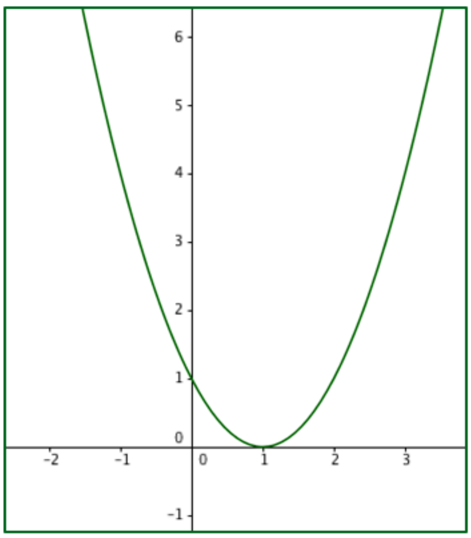
We have met one of these already this week (2.4) with our dots pattern. These functions have a highest power of 1 on the variable. For example:



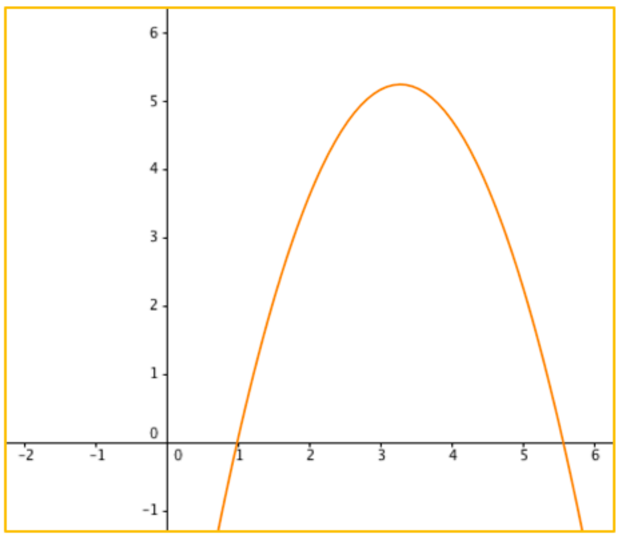




### Quadratic:

You will have met plenty of these at Junior Cycle. This are functions that have a highest power of 2 on the variable. For example:



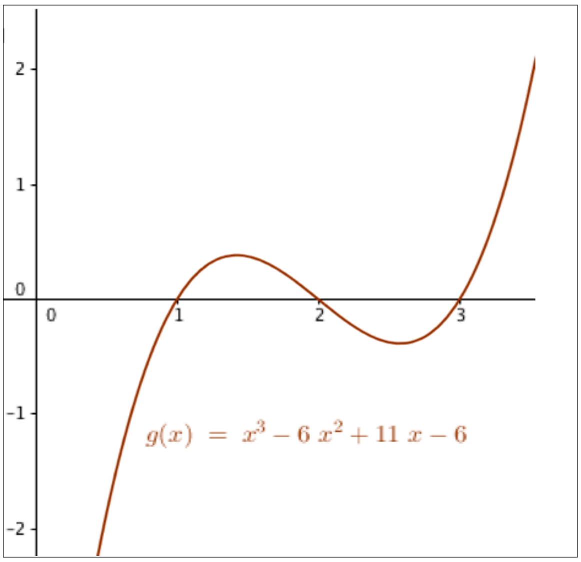
$h(x)$has a **positive** $x^2$



$j(x)$ has a **negative** $x^2$

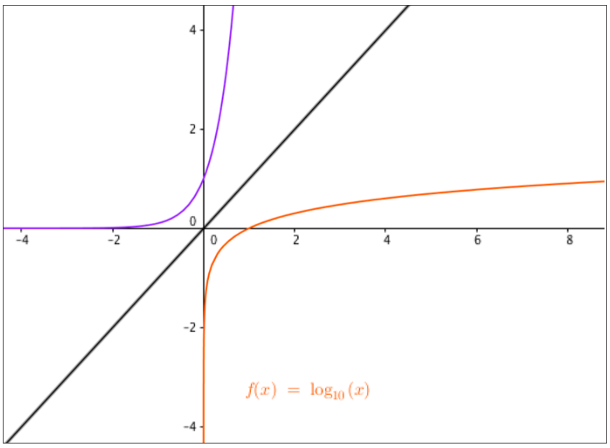
## Cubic:

You may have seen some of these at Junior Cycle. For a cubic function, the highest power on the variable will be 3. For example:




## Exponential & Logarithmic:

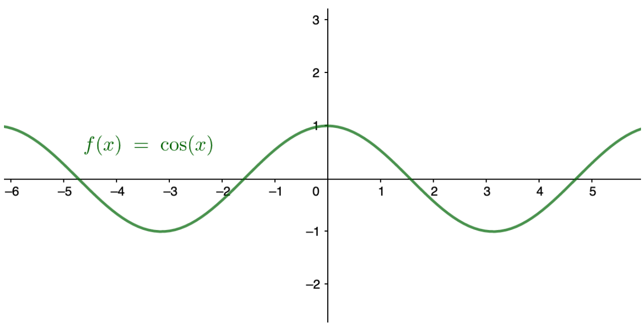
These functions are more than likely new to you at Senior Cycle. For an exponential function, the variable will be the power, for example $f(x) = 10^x$. For a logarithmic function you will see a log in the function, for example $g(x) = \log_{10} x$. Graphically, these functions look like this:




*these two functions mirror one another through the line

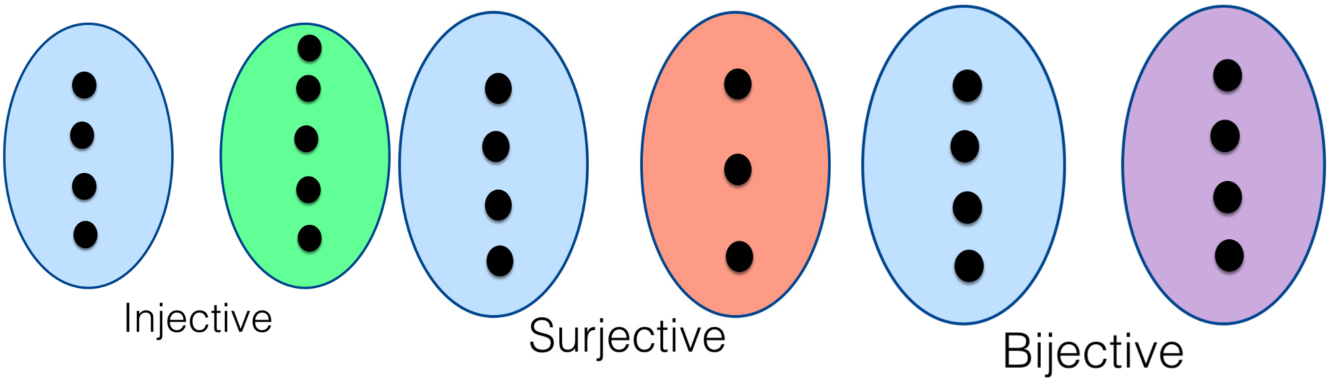
## Trigonometric:

Any function containing one of the trigonometric operators such as sin, cos, tan etc. is of this type.



## 2.8 Characteristics of Functions

In addition to the above types of functions, we can look at the nature of these functions and describe them as injective, surjective or bijective. To look at this in more detail complete the diagrams below in conjunction with the video for Week 2.



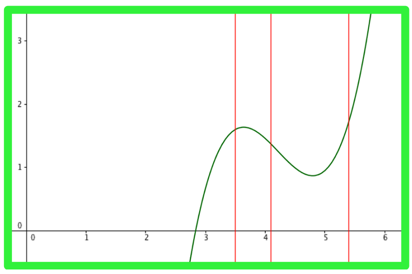

**Injective:**

**Surjective:**

**Bijective:**

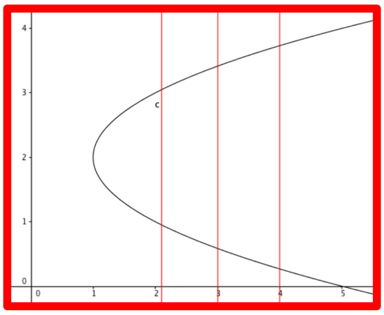
### Vertical Line Test for a function:

Using this test, we draw a vertical line through our function, if the line cuts the function ONLY once then we have a function.



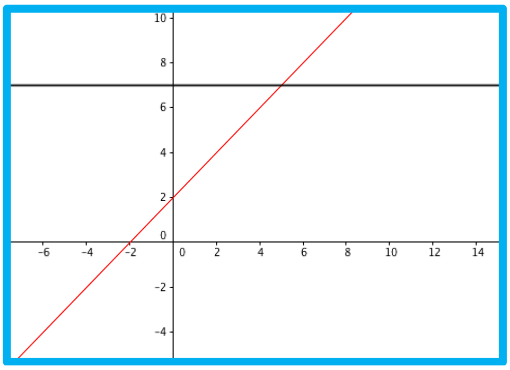
However...

If the vertical line cuts the graph more than once then the graph is not a function.



**Horizontal Test for Injectivity:**

We can use a horizontal line to test a function for injectivity. Just like the previous test, if the line cuts the function at only one point, then we can say the function is injective (one-to-one).



## 2.9 Inverse of Functions

In mathematics we are often looking to reverse a process. For example, addition and subtraction, multiplication and division and so on. So, when it comes to functions, we are also interested in undoing. If $f(x)$ is a function, then we denote its inverse with $f^{-1}(x)$.

$f(x) = 2x + 1$

Which makes its inverse function:

$f^{-1}(x) = \frac{x-1}{2}$

Let's have a look at how we got there:

How does that look graphically? Be sure to use your graphing software of choice to check this out.

## 2.10 Recap of Learning Intentions

**After this week's lesson you will be able to;**

- Generalise a sequence
- Describe the nature of a sequence
- Represent a sequence in multiple ways
- Describe functions in terms of injective, surjective and bijective

## 2.11 Homework Task

Label the following functions as Injective, Surjective or Bijective. Be sure to use your graphing software to help with visualizing the functions. Be sure to write down the reason you have chosen your label. Be careful, one or two of these are tricky.

$f(x) = x + 3$

$g(x) = x^2 + 5x + 6$

$h(x) = \frac{1}{x+1}$

$x^2 + y^2 = 25$

$j(x) = \frac{x^2+5x+6}{x+2}$

## 2.12 Solution to 1.11 Task

Tick ✓ for yes or cross ✗ for whether a particular number belongs to the appropriate number system.

| Number | $\mathbb{N}$ | $\mathbb{Q}$ | $\mathbb{R}$ | $\mathbb{Z}$ | $\mathbb{R} \setminus \mathbb{Q}$ |
|---|---|---|---|---|---|
| 9 | ✓ | ✓ | ✓ | ✓ | ✗ |
| $\sqrt{5}$ | ✗ | ✗ | ✓ | ✗ | ✓ |
| -2 | ✗ | ✓ | ✓ | ✓ | ✗ |
| 8.5 | ✗ | ✓ | ✓ | ✗ | ✗ |
| $\frac{2\pi}{5}$ | ✗ | ✗ | ✓ | ✗ | ✓ |
| $\sqrt{16}$ | ✓ | ✓ | ✓ | ✓ | ✗ |